NUMBER SYSTEM

Number line :→

A line with numbers placed in their Correct position, useful for adition and subtraction and for Showing relations between numbers.

Natural numbers :→

Counting numbers are known as natural numbers. It is denoted by N. eg 1, 2, 3, 4----

Whole numbers :—

A whole number is a member of the set {0,1,2,3}. It is either one of the positive integers (natural number) or zero. example :- 0, 1, 2, 3----

Integers:→

An integer is a whole number that can be positive negative or zero. example −2, −1, 0, 1, 2 ---

Rational number :→

A rational number is any number that Can be expressed in the form P/q of two integers, a numerator P and a non-zero denominator. q

Real number :→

The type of numbers we normally use such as 1, 15.82, −0.1, 3/4 etc. Positive or negative, large or small whole number or decimal numbers are all Real numbers

Note :→ The decimal expansion of rational number is either terminating or non terminating recurring. Moreover a number whose decimal expansion is terminating or non terminating recurring is rational e.g 0.3333 - - - -.

The decimal expansion of an irrational number is non terminating non-recuring. Moreover, a number whose decimal expansion is non-terminating non recurring is irrational.

Exercise 1.1

Q 1. Is zero a rational number? Can you write it in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$

Solution :→ Yes zero is a rational number because we can write it in the form $\frac{0}{1}, \frac{0}{2}$ ---

where $q \neq 0$

Q 2 find six rational numbers between 3 and 4

Sol :→ $3 = \frac{3 \times 8}{8} = \frac{24}{8}$

and $4 = \frac{4 \times 8}{8} = \frac{32}{8}$

Hence rational numbers between 3 & 4 are as

$\frac{25}{8}, \frac{26}{8}, \frac{27}{8}, \frac{28}{8}, \frac{29}{8}, \frac{30}{8}$

Q 3 find five rational number between $\frac{3}{5}$ and $\frac{4}{5}$

Sol :→ $\frac{3}{5} = \frac{3 \times 6}{5 \times 6} = \frac{18}{30}$

and $\frac{4}{5} = \frac{4 \times 6}{5 \times 6} = \frac{24}{30}$

Now rational number btw $\frac{3}{5}$ & $\frac{4}{5}$ are

$\frac{19}{30}, \frac{20}{30}, \frac{21}{30}, \frac{22}{30}, \frac{23}{30}$

Q4 State wheather the following statements are true or false Give reasons for your answers.

1) Every natural number is a whole number

Sol Yes, because natural numbers starts from 1 ie (1, 2, 3 --) and whole numbers starts from 0 ie (0, 1, 2, 3, 4 ---) as from above examples we can see that every natural number is a whole number.

2) Every integer is a whole number

Sol No, because whole number starts from 0 ie (0, 1, 2, ---) but integers are +ve and negative of all natural numbers including zero ie ---−2, −1, 0, 1, 2 ---

3) Every rational number is a whole number.

Sol No, whole number does not contain fractions eg $\frac{1}{2}$ is a rational number but not a whole number.

EXERCISE 1.2

Q1. State whether the following statements are true or false. Justify your answers.

i) Every rational number is a real number

Sol :→ True, as collection of real numbers contain both rational and irrational numbers.

ii) Every point on the number line is of form $\sqrt{m}$, where m is a natural number

Sol : → False. As no negative number can be square root of any number

iii) Every real number is an irrational number.

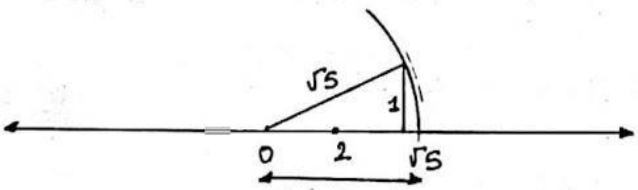
Sol : → False. as 5 is real but not irrational.

Q2. Are the square roots of all positive integers irrational? if not give an example of the square root of a number that is a rational number.

Sol :→ False as $\sqrt{4} = 2$ is a rational number.

Q3 Show how $\sqrt{\ }$ can be represented on a number line

Sol To represent $\sqrt{5}$ on the number line we take a length of



two units from 0 on the number line in positive direction and one unit perpendicular to it. The hypotenuse of the triangle thus formed is $\sqrt{5}$. Then with the help of a divider a length equal to the hypotenuse of $\sqrt{5}$ units can be cut on the number line

EXERCISE :→ 1.3

Q1. Write the following in decimal form and say what kind of decimal expansion each has.

i) $\frac{36}{100} = 0.36$, Hence terminating

ii) $\frac{1}{11} = 0.\overline{09}$. Hence recuring, non terminating

iii) $4\frac{1}{8} = \frac{33}{8} = 4.125$, terminating

iv) $\frac{3}{13} = 0.\overline{230769}$, recurring non terminating

v) $\frac{2}{11} = 0.\overline{18}$, non terminating recuring.

vi) $\frac{329}{400} = 0.8225$, terminating.

Q2. You know that $\frac{1}{7} = 0.\overline{142857}$. Can you predict what the decimal expansion of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$ are, without actual doing the long division.

Sol:→ As $\frac{2}{7} = 2 \times \frac{1}{7} = 2 \times 0.\overline{142857} = 0.\overline{285714}$

Similarly $\frac{3}{7} = 3 \times \frac{1}{7} = 0.\overline{428571}$

$\frac{6}{7} = 6 \times \frac{1}{7} = 0.\overline{857142}$

$\frac{4}{7} = 4 \times \frac{1}{7} = 0.\overline{571428}$

$\frac{5}{7} = 5 \times \frac{1}{7} = 0.\overline{714285}$

Q3 Express the following in the $\frac{p}{q}$ where p and q are integers and $q \neq 0$

i) $0.\overline{6}$

Sol:→ Let $x = 0.6666\ldots$ —— (i)

Multiply eq (i) by 10 we get

$10x = 6.6666\ldots$ —— (ii)

Sub (i) from (ii)

$9x = 6$

$x = \frac{6}{9} = \frac{2}{3}$

$\boxed{\therefore \; x = \frac{2}{3}}$

ii) Let $x = 0.4\overline{7} = 0.47777\ldots$ —(i)

Multiply eq (i) by 10 we get

$10x = 4.77777\ldots$ (ii)

Sub (i) from (ii)

$9x = 4.3$

$\Rightarrow x = \frac{4.3}{9} = \frac{43}{90}$ Ans

$\Rightarrow \boxed{x = \frac{43}{90}}$

iii) $x = 0.\overline{001}$ —(i)

Multiply eq (i) by 1000 we get

$1000x = 1.\overline{001}$ —— (ii)

Sub (i) from (ii) we get

$999x = 1$

$\Rightarrow x = \frac{1}{999}$

Hence $\boxed{x = \frac{1}{999}}$

Q4 Express 0.9999 in the form $\frac{p}{q}$, Are you surprised by your answer? with your teachers and classmates discuss why the answer makes sense.

Sol :→ Let $x = 0.9999\ldots$ —(i)

Multiply b/s by 10 we get

$10x = 9.9999\ldots$ (ii)

Sub (i) from (ii) we get

$9x = 9$

$\Rightarrow x = \frac{\cancel{9}}{\cancel{9}} = 1$

The answer makes sense as $0.\overline{9}$ is infinitely close to 1.

Q5 What can the maximum number of digits be in the repeating blocks of digit in the decimal expansion of $\frac{1}{17}$? Perform the division to check your answer.

Sol :→ The maximum number of digit in repeating block is $16 (< 17)$

Division gives $\frac{1}{17} = 0.\overline{0588235294117647}$

The repeating block has 16 digit Ans//

Q6 Look at several examples of rational number in the form $\frac{p}{q}$ $(q \neq 0)$ where p and q are integers with no common factor other than 1 and having terminating decimal expansion. Can you guess what property q must satisfied?

Sol :→ $\frac{2}{5} = 0.4$, $\frac{3}{2} = 1.5$, $\frac{7}{8} = 0.875$, $\frac{7}{10} = 0.7$

All the denominator are either 2 (or its power), 5 (or its power) or a combination of both

Q7 Write three numbers whose decimal expansion are non-terminating non-recuring.

Sol :→ 7.31411411114 - - -

0.10100200030000 4 - -

$\pi = 3.1416$ - - - -

Q8 find three different irrational number btw $\frac{5}{7}$ and $\frac{9}{11}$

Sol :→ $\frac{5}{7} = 0.\overline{714285}$

$\frac{9}{11} = 0.\overline{81}$

There are infine number of irrational numbers btw these two numbers we may chose any three of them.

0.7234596
0.7425735
0.78123957

Q9 Classify the following numbers as rational or irrational

i) $\sqrt{23}$

Sol Irrational

ii) $\sqrt{225} = 15$ = rational

iii) 0.3796, rational because terminating

iv) 7.478478 - - - Irrational

v) 1.101001000l - - - Irrational.

EXERCISE 1.4

Q1 Visualise 3.765 on the number line, using successive magnification. www.ndjtuition.com

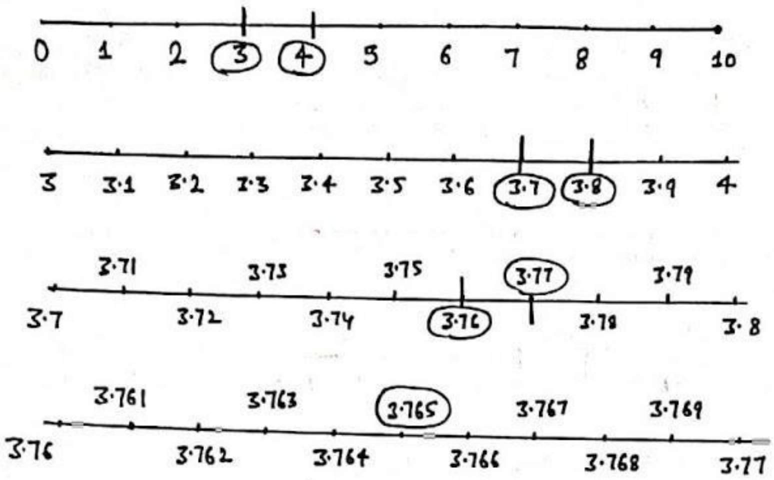
Sol :→ Draw a number line

Magnify the interval btw 3 and 4 and divide it into 10 equals parts

Now magnify the interval btw 3.7 and 3.8 into 10 equal parts, the number lies btw 3.76 and 3.77

Magnify the interval between 3.76 and 3.77 and divide it into 10 equal parts

Now 3.675 is the fifth division in this magnification.



Q2 Visualise $4.\overline{26}$ on the number line, up to 4 decimal places.

Sol :→ Step one - on the number line the given number $4.\overline{26}$ lies between 4 and 5 (For four decimal places number is 4·2626)

Step two :→ Magnify the interval 4 and 5 and divide it into 10 equal parts.

step three :→ The given number 4·2626 lies between 4·2 and 4·3

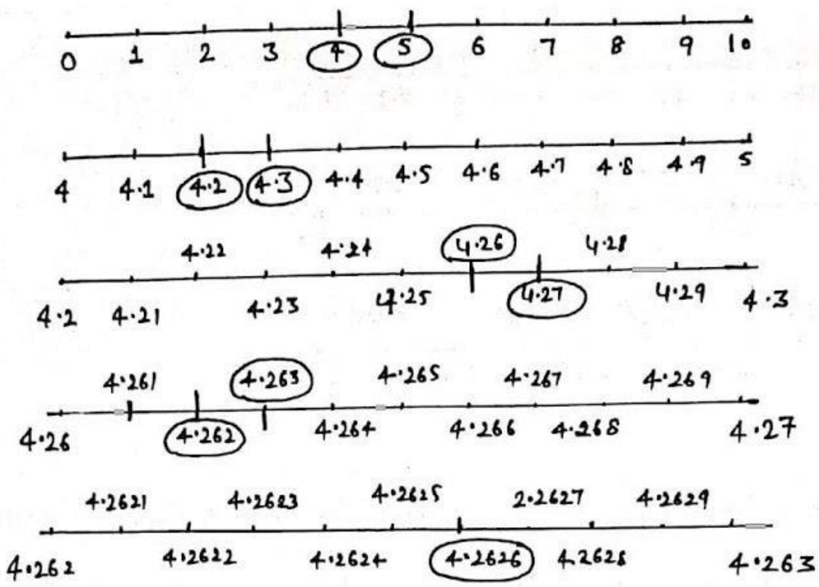
Step four :→ Magnify the interval between 4·2 and 4·3 and divide it into ten equal parts.

step five :→ The given number falls between 4·26 and 4·27

Step Six :→ Magnify 4·26 and 4·27 and divide it into ten equal parts.

Step seven :→ Magnify 4·262 and 4·263 and divide it into ten equal parts

Step nine :→ The given number is the sixth division of given interval.


0 1 2 3 4 5 6 7 8 9 10
4 4.1 4.2 4.3 4.4 4.5 4.6 4.7 4.8 4.9 5
4.22 4.24 4.26 4.28
4.2 4.21 4.23 4.25 4.27 4.29 4.3
4.261 4.263 4.265 4.267 4.269
4.26 4.262 4.264 4.266 4.268 4.27
4.2621 4.2623 4.2625 2.2627 4.2629
4.262 4.2622 4.2624 4.2626 4.2628 4.263

EXERCISE 1.5

Q1 Classify the following number as rational or irrational.

i) $2-\sqrt{5}$. rational

ii) $3+\cancel{\sqrt{23}}-\cancel{\sqrt{23}} = 3$, rational

iii) $\dfrac{2\cancel{\sqrt{7}}}{7\cancel{\sqrt{7}}} = \dfrac{2}{7}$ rational

iv) $\dfrac{1}{\sqrt{2}} =$ irrational

v) $2\pi = 2 \times 3.14\ldots$ $\therefore$ is irrational

Q2 Simplyfy each of the following expression.

i) $(3+\sqrt{3})(2+\sqrt{2})$

Sol $3(2+\sqrt{2}) + \sqrt{3}(2+\sqrt{2})$

$= 6 + 3\sqrt{2} + 2\sqrt{3} + \sqrt{6}$ Ans

ii) $(3+\sqrt{3})(3-\sqrt{3})$

Sol $(3)^2 - (\sqrt{3})^2$ using $(a+b)(a-b) = a^2-b^2$

$9-3 = 6$ Ans

iii) $(\sqrt{5}+\sqrt{2})^2$

Sol $(\sqrt{5})^{\cancel{2}} + (\sqrt{2})^{\cancel{2}} + 2\sqrt{5}\sqrt{2}$ using $(a+b)^2 = a^2+b^2+2ab$

$= 5+2+2\sqrt{10}$

$= 7+2\sqrt{10}$. Ans//

iv) $(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})$

Sol $(\sqrt{5})^{\cancel{2}} - (\sqrt{2})^{\cancel{2}}$ Using $(a-b)(a+b) = a^2-b^2$

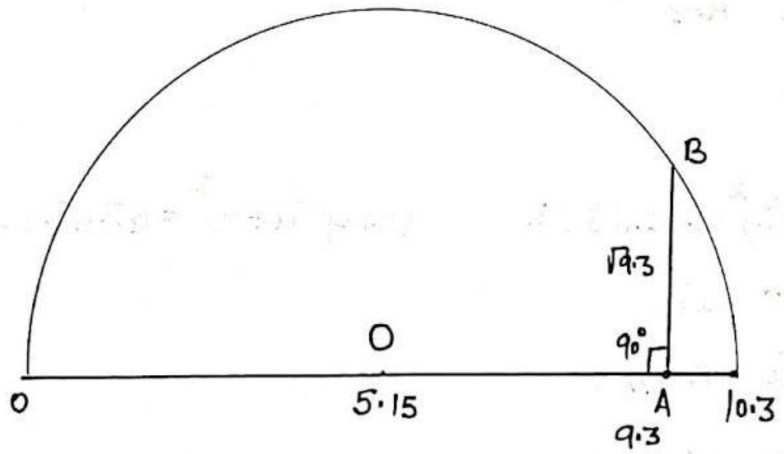
$5-2 = 3$ Ans//

Q3 Recall, π is defined as ratio of the Circumference of a circle to its diameter (say d) That is $\pi = \frac{c}{d}$ This seems to contradict the facts that π is irrational, How will you resolve this contradiction?

Ans :→ with a scale or tape we get only an approximate rational number as the reasult of our measurement. That is why π can be approximately represented as a quotient of two rational numbers. As a result of mathematical truth it is irrational.

Q4 Represent $\sqrt{9.3}$ on the number line.

Sol:→ To represent $\sqrt{9.3}$ draw a segment of 9.3 units on the number line. Let A represent 9.3. Extend it by 1 cm. Show point $\frac{10.3}{2} = 5.15$ by on the number line. With 'O' as Centre and radius 5.15 units, draw a semicircle. Draw AB perpendicular to OA to cut the hemisphere at B. The length AB is $\sqrt{9.3}$ units.



Q5 Rationalise the denominators of the following.

1) $\frac{1}{\sqrt{7}} = \frac{1}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{7}}{7}$ Ans

2) $\frac{1}{\sqrt{7}-\sqrt{6}} = \frac{1}{\sqrt{7}-\sqrt{6}} \times \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}} = \frac{\sqrt{7}+\sqrt{6}}{(\sqrt{7})^{\cancel{2}}-(\sqrt{6})^{\cancel{2}}} = \frac{\sqrt{7}+\sqrt{6}}{1}$ Ans//

3) $\frac{1}{\sqrt{5}+\sqrt{2}} = \frac{1}{\sqrt{5}+\sqrt{2}} \times \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}} = \frac{\sqrt{5}-\sqrt{2}}{(\sqrt{5})^{\cancel{2}}-(\sqrt{2})^{\cancel{2}}} = \frac{\sqrt{5}-\sqrt{2}}{3}$ Ans//

4) $\frac{1}{\sqrt{7}-2} = \frac{1}{\sqrt{7}-2} \times \frac{\sqrt{7}+2}{\sqrt{7}+2} = \frac{\sqrt{7}+2}{7-4} = \frac{\sqrt{7}+2}{3}$ Ans//

EXERCISE 1.6

Q1. Find

i) $64^{1/2} = (8^{\cancel{2}})^{1/\cancel{2}} = 8$ Ans

ii) $32^{1/5} = (2\times2\times2\times2\times2)^{1/5} = ((2)^{\cancel{5}})^{1/\cancel{5}} = 2$ Ans

iii) $125^{1/3} = ((5)^{\cancel{3}})^{1/\cancel{3}} = 5$ Ans//

iv) $125^{-1/3} = \frac{1}{(125)^{1/3}} = \frac{1}{((5)^{\cancel{3}})^{1/\cancel{3}}} = \frac{1}{5}$ Ans

Q2 Simplyfy :→

i) $9^{3/2} = ((9)^{1/2})^3 = (3)^3 = 27$

Or $((3)^{\cancel{2}})^{3/\cancel{2}} = (3)^3 = 27$ Ans

ii) $(32)^{2/5} = ((2)^{\cancel{5}})^{2/\cancel{5}} = (2)^2 = 4$ Ans//

iii) $16^{3/4} = ((2)^{\cancel{4}})^{3/\cancel{4}} = (2)^3 = 8$ Ans

iv) See Q1 Part (iv)

Q3 Simplyfy ! →

i) $2^{2/3} \cdot 2^{1/5}$

Sol $2^{2/3} \cdot 2^{1/5}$ Base Same power added

$(2)^{2/3+\frac{1}{5}} = (2)^{\frac{13}{15}}$ Ans

ii) $\left(\frac{1}{3^3}\right)^7 = (3^{-3})^7 = (3)^{-21}$ Ans

iii) $\frac{11^{1/2}}{11^{1/4}} = 11^{1/2} \cdot 11^{-1/4} = (11)^{\frac{1}{2}-\frac{1}{4}} = (11)^{1/4}$ Ans//

iv) $7^{1/2} \cdot 8^{1/2} = (56)^{1/2}$ Ans (Power same So, base multiplied)

Logarithm

Definition and laws of Logarithms :→

If for a positive real number a, $a \neq 1$, $a^m = b$. where m is rational and b is a real number, the m is said to be logarithm of b to tha base a. we write this as $\log_a b = m$

"log" being the abbrevation of the word 'logarithm'

For example :→

$\log_2 8 = 3$ Since $2^3 = 8$

$\log_2 16 = 4$ Since $2^4 = 16$

$\log_3 1 = 0$ since $3^0 = 1$

Logarithm to the base 10 :→

Since the number 10 is the base of writing number, It is Convenient to use 10 as the base to the Logarithm.

Examples :→

i) $\log_{10} 10 = 1$ Since $10^1 = 10$

ii) $\log_{10} 1 = 0$ Since $10^0 = 1$

iii) $\log_{10} 100 = 2$ Since $10^2 = 100$

iv) $\log_{10}(0.01) = -2$ since $10^{-2} = \frac{1}{100} = 0.01$

v) $\log_{10}(0.01) = -3$ Since $10^{-3} = \frac{1}{1000} = 0.001$

Laws of Logarithm

First law For any base $(a>0), a\neq 1)$

$$\log_a mn = \log_a m + \log_a n$$

Proof Let $\log_a m = x$, and $\log_a n = y$ then $a^x = m$ and $a^y = n$

we have $mn = a^x a^y = a^{x+y}$ or $a^{x+y} = mn$

By the defination of logarithm, it follows that

$$\log_a (mn) = x+y = \log_a m + \log_a n$$

Second Law: $\longrightarrow$ For any base $(a>0, a\neq 1)$

$$\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$$

Proof: $\rightarrow$ Let $\log_a m = x$ and $\log_a n = y$ so that $a-m$

and $a^y = n$ now $\frac{m}{n} = \frac{a^x}{a^y} = a^{x-y}$ or $a^{x-y} = \frac{m}{n}$

By the definition of Logarithm, we have

$$\log_a\left(\frac{m}{n}\right) = x-y = \log_a m - \log_a n$$

Third Law: $\longrightarrow$ For any base $(a>0, a\neq 1)$

$$\log_a (m)^n = n\log_a m$$

Proof :→ Let $\log_a m = x \Rightarrow a^x = m$

Now $m^n = (a^x)^n = a^{nx}$ ie $a^{nx} = m^n$

Again by using the definition of logarithm we have

$\log_a (m)^n = nx = n \log_a m$.

Examples :→

1) $\log 12 = \log(3 \times 4) = \log 3 + \log 4$ (By first Law)

2) $3 \log 2 + \log 5 = \log 2^3 + \log 5 = \log 8 + \log 5$

$\Rightarrow \log(8 \times 5) = \log 40$

EXERCISE 1.7

Q1 Write the following in the form of logarithms.

i) $2^5 = 32$

Sol :→ $\log_2 32 = 5$

ii) $5^5 = 3125$

Sol $\log_5 3125 = 5$

iii) $10^{-1} = 0.1$

Sol :→ $\log_{10} 0.1 = -1$

Q 2, Write the following in exponential form

i) $\log_3 243 = 5$

Sol $3^5 = 243$

ii) $\log 10.1000 = 3$

Sol ~~to~~ $10^3 = 1000$

iii) $\log_{10}(0.0001) = -4$

Sol $10^{-4} = 0.0001$

Q 3 For any base a $(a>0,\ a \neq 1)$ Prove that

$$\log a(mnp) = \log a m + \log_a n + \log a p$$

Sol :→ let $\log_a m = x$ So. $a^x = m$

$\log_a n = y$ $a^y = n$

$\log_a p = z$ $a^z = p$

∴ we have $mnp = a^x \cdot a^y \cdot a^z = a^{x+y+z} = mnp$

So by defination of logarithm

$$\log a(mnp) = x+y+z$$

$$\Rightarrow \log_a(mnp) = \log_a m + \log_a n + \log_a p$$

4) In each of the following assume that the base is 10. Prove that:

i) $\log\left(\frac{1}{2}\right) + \log\left(\frac{2}{3}\right) + \log\left(\frac{3}{4}\right) + \log\left(\frac{4}{5}\right) - \log\left(\frac{1}{5}\right) = 0$

<u>Sol</u> Using $\log \frac{m}{n} = \log m - \log n$ we get

$$= \log 1 - \log 2 + \log 2 - \log 3 + \log 3 - \log 4 + \log 4 - \log 5$$

$$- \log 1 + \log 5 = 0$$

$\Rightarrow \quad 0 = 0$

Hence L.H.S = R.H.S Proved //

ii) $\log 360 = 3\log 2 + 2\log 3 + \log 5$

<u>Sol</u> $\log 360 = \log 2^3 + \log 3^2 + \log 5$

(using $m \log n = \log n^m$)

$\Rightarrow \log 360 = \log 8 + \log 9 + \log 5$

$\Rightarrow \log 360 = \log (8 \times 9 \times 5)$

$\Rightarrow \log 360 = \log 360$

Hence proved //

iii) $\log\left(\frac{50}{147}\right) = \log 2 + 2\log 5 - \log 3 - 2\log 7$

Sol $\log\left(\frac{50}{147}\right) = \log 2 + \log 5^2 - \log 3 - \log 7^2$

$\log\left(\frac{50}{147}\right) = \log 2 + \log 25 - \log 3 - \log 49$

$\log\left(\frac{50}{147}\right) = (\log 2\times 25) - (\log 3\times 49)$

$\log\left(\frac{50}{147}\right) = \log 50 - \log 147$

$\log \frac{50}{147} = \log \frac{50}{147}$

Hence proved

iv) $\log 10 + \log 100 + \log 1000 + \log 10000 = 10$

Sol Using $\log m + \log n = \log mn$

$\log(10\times 100\times 1000\times 10000) = 10$

$= \log 10000000000 = 10$

$\Rightarrow \log_{10} 10000000000 = 10$

$\Rightarrow 10 = 10$

Since

$\log_{10} 10 = 1$

$\log_{10} 100 = 2$

Hence proved

v) $5 \log 3 - \log 9 = \log 27$

Sol $\log 3^5 - \log 9 = \log 27$

$\log 243 - \log 9 = \log 27$

$\Rightarrow \log\left(\frac{\cancel{243}}{\cancel{9}}\right) = \log 27$

$\Rightarrow \log 27 = \log 27$

Hence proved.